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Energy-Momentum of a Cosmological Brane Model and the Gauge Hierarchy

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Abstract We analyze in this paper the general covariant energy-momentum tensor of the gravitational system in general five-dimensional cosmological brane-world models. Then through calculating this energy-momentum for the cosmological generalization of the Randall-Sundrum model, which includes the original RS model as the static limit, we are able to show that the weakness of the gravitation on the "visible" brane is a general feature of this model. This is the origin of the gauge hierarchy from a gravitational point of view. Our results are also consistent with the fact that a gravitational system has vanishing total energy.

Keywords Energy-momentum · Brane · Hierarchy problem

1 Introduction

Since the early attempts of Einstein [1] and others [2–6], the problem of the conservation and localization of the energy-momentum of the gravitational field have been a highly controversial question for decades. These works share a common disadvantage that their expressions of the energy-momentum of the gravitational field are all in the form of pseudotensors, which are not covariant objects because they inherently depend on the reference frame thus cannot provide a truly physical local gravitational energy-momentum density. This difficulty comes from the fact that gravitational energy-momentum density cannot be locally defined because of the equivalence principle [7], which states that gravitational field should

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not be detectable at a specific point. Duan [8] has proposed a generally covariant form of the conservation law of energy-momentum using the orthonormal frames, in which the energy-momentum is a covariant vector in Riemannian spacetime. It is generally covariant and is able to overcome the flaws in the expressions from Einstein and others. There are many advantages of this form of energy-momentum [9–11] and we will use it in our calculations.

The gauge hierarchy problem, i.e. the weakness of the gravitational interaction compared to other interactions, is one of the most fundamental problems in theoretical physics nowadays. Randall and Sundrum proposed a model [12, 13] with one extra dimension which is compactified as $S^1/\mathbb{Z}_2$ with an exponential "warped" factor in the metric. They argue that the gauge hierarchy comes from the fact that the gravitational field is localized at the "hidden" or Planck brane and is very weak at our "visible" or TeV brane due to the warped factor. There are numerous works on the generalization and phenomenological applications of the RS model, such as the generalization to cosmological issues [14–17]. There is also a discussion about the energy-momentum of RS model by Liu and others [18]. Our argument is a generalization of the discussion in [18] and is mostly based on the cosmological brane-world models.

This paper is arranged as following. In Sect. 2 we first describe the setup of our work which includes one single extra dimension, then we briefly review some of the features of the generally covariant form of the conservation law of energy-momentum proposed by Duan. After that we use this formulation to calculate the energy-momentum of our general setup. In Sect. 3 we calculate the energy-momentum of one particular cosmological brane model which is a generalization of the original RS model, and discuss the results in order to show that the weakness of the gravitational field on our brane is a general feature of this model. Finally, we discuss the implications of our results.

2 Energy-Momentum from a Five-Dimensional Spacetime

We start with a five-dimensional spacetime with two 3-branes in it. The fifth dimension which is labeled as y is compactified as $S^1/\mathbb{Z}_2$. The two 3-branes locate at the two fixed points of the orbifold $y=0$ and $y=\pi$. The total action of this system is then

$$S=\int d^4x dy\sqrt{-g}\,[2M^3R-\Lambda]+\sum_{i=1,2}\int d^4x\sqrt{-g^{(i)}}[\mathcal{L}_i-\Lambda_i] \tag{1}$$

where g_{MN} and R denote the five-dimensional metric and Ricci scalar respectively, Λ and Λ_i are the cosmological constants of that bulk and the branes, and $g^{(i)}_{\mu\nu}$ is the induced metric on the branes. The uppercase Latin letters M, N stand for the five-dimensional indices. The signature of g_{MN} is $(-++++)$. We have separated the gravitational part and the matter part of the action. The general five-dimensional metric from a cosmological point of view, which means that the 3-branes should be spatially homogeneous and isotropic, is then

$$ds^2=-n^2(t,y)dt^2+a^2(t,y)\delta_{ij}dx^i dx^j+b^2(t,y)dy^2 \tag{2}$$

where we have assumed for simplicity that the usual three-dimensional space are also spatially flat. The Einstein tensors are [14–16]

$$G_{00}=3\left[\frac{\dot a}{a}\left(\frac{\dot a}{a}+\frac{\dot b}{b}\right)-\frac{n^2}{b^2}\left[\frac{a''}{a}+\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)\right]\right] \tag{3}$$

$$G_{ii} = \frac{a^2}{b^2}\left[\frac{a'}{a}\left(\frac{a'}{a}+2\frac{n'}{n}\right)-\frac{b'}{b}\left(\frac{n'}{n}+2\frac{a'}{a}\right)+2\frac{a''}{a}+\frac{n''}{n}\right]$$
$$+\frac{a^2}{n^2}\left[\frac{\dot a}{a}\left(-\frac{\dot a}{a}+2\frac{\dot n}{n}\right)-2\frac{\ddot a}{a}+\frac{\dot b}{b}\left(-2\frac{\dot a}{a}+\frac{\dot n}{n}\right)-\frac{\ddot b}{b}\right] \tag{4}$$

$$G_{04} = 3\left(\frac{n'}{n}\frac{\dot a}{a}+\frac{a'}{a}\frac{\dot b}{b}-\frac{\dot a'}{a}\right) \tag{5}$$

$$G_{44} = 3\left[\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right)-\frac{b^2}{n^2}\left[\frac{\dot a}{a}\left(\frac{\dot a}{a}-\frac{\dot n}{n}\right)+\frac{\ddot a}{a}\right]\right] \tag{6}$$

Now we turn our attention to the formulation of the energy-momentum tensor. Here we will use the form proposed by Duan and others [8–11]. According to their argument, the total energy-momentum tensor of the gravitational system can be derived from the Lagrangian of the gravity in a gravitational theory described by the orthonormal frames, i.e. the vielbeins. Let's think about a field theory in n dimensional spacetime with the action

$$I = \int d^n x \mathcal{L}(\phi^A, \partial_\mu \phi^A) \tag{7}$$

where ϕ^A represents general fields. If the action is invariant under a infinitesimal transformations

$$x \to x^{\mu'} = x^\mu + \delta x^\mu \tag{8}$$

$$\phi^A(x) \to \phi'^A(x') = \phi^A(x) + \delta\phi^A(x) \tag{9}$$

where we use the Greek letters like μ to denote the spacetime coordinates for clarity. If we assume that $\delta\phi^A(x)$ vanishes on the boundary of the spacetime manifold, the it can be proven that the following result is true [8–11]

$$\partial_\mu\left(\mathcal{L}\delta x^\mu + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi^A}\delta_0\phi^A\right) + [\mathcal{L}]_{\phi^A}\delta_0\phi^A = 0 \tag{10}$$

where $\delta_0\phi^A$ is the Lie derivative of ϕ^A

$$\delta_0\phi^A = \phi'^A(x) - \phi^A(x) = \delta\phi^A(x) - \partial_\mu\phi^A\delta x^\mu \tag{11}$$

while $[\mathcal{L}]_{\phi^A}$ is

$$[\mathcal{L}]_{\phi^A} = \frac{\partial\mathcal{L}}{\partial\phi^A} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^A} \tag{12}$$

If $\mathcal{L}$ is the total Lagrangian of the system, the we have $[\mathcal{L}]_{\phi^A} = 0$ as the field equation of the field ϕ^A. Then from (10) we can get a conservation law corresponding to infinitesimal transformations (8) and (9)

$$\partial_\mu\left(\mathcal{L}\delta x^\mu + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi^A}\delta_0\phi^A\right) = 0 \tag{13}$$

However, if $\mathcal{L}$ is not the total Lagrangian of the system, although (10) will still hold, (12) will not.

If we use the vielbein field as the fundamental field in gravitational theory, the we can decompose ϕ^A as $\phi^A = (e^\mu_a, \psi^B)$, in which ψ^B is an arbitrary tensor under general displacement transformations and e^μ_a denotes the vielbeins. ψ^B can always be transformed to a scalar using vielbeins [8–11], so we can eliminate them from the gravitational Lagrangian $\mathcal{L}_g$, so that (10) becomes

$$\partial_\mu\left(\mathcal{L}\delta x^\mu + \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^\nu_a}\delta_0 e^\nu_a\right) + [\mathcal{L}]_{e^\nu_a}\delta_0 e^\nu_a = 0 \tag{14}$$

while from the infinitesimal transformations (8) and (9) the Lie derivative becomes

$$\delta_0 e^\mu_a = e^\mu_a \delta(\partial_\nu x^\mu) - (\partial_\nu e^\mu_a)\delta x^\nu \tag{15}$$

so we can get

$$\partial_\mu\left[\left(\left(\mathcal{L}_g\delta^\mu_\sigma - \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^\nu_a}\partial_\sigma e^\nu_a\right) + [\mathcal{L}_g]_{e^\sigma_a} e^\mu_a\right)\delta x^\sigma + \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^\nu_a} e^\sigma_a \delta(\partial_\sigma x^\nu)\right] = 0 \tag{16}$$

which is a general conservation law. If we define

$$I^\mu_\sigma = \mathcal{L}_g\delta^\mu_\sigma - \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^\nu_a}\partial_\sigma e^\nu_a + [\mathcal{L}_g]_{e^\sigma_a} e^\mu_a \tag{17}$$

$$V^{\mu\sigma}_\nu = \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^\nu_a} e^\sigma_a \tag{18}$$

then the conservation law (16) becomes

$$\partial_\mu[I^\mu_\sigma\delta x^\sigma + V^{\mu\sigma}_\nu\delta(\partial_\sigma x^\nu)] = 0 \tag{19}$$

The conservation of energy-momentum in special relativity is from the invariance of the action under the infinitesimal transformation of the Lorentz coordinates. It can be generated to the general relativity case as being the general displacement transformation [8–11]

$$x'^\mu = x^\mu + \delta x^\mu \qquad \delta x^\mu = e^\mu_a b^a \tag{20}$$

then the general conservation law above means

$$\partial_\mu(I^\mu_\sigma e^\sigma_a + V^{\mu\nu}_\sigma\partial_\nu e^\sigma_a) = 0 \tag{21}$$

From Einstein equation $\sqrt{-g}T^\mu_a = [\mathcal{L}_g]_{e^a_\mu}$, where T^μ_a is the energy-momentum of matter fields, we have this relation

$$I^\mu_\nu e^\nu_a = \left(\mathcal{L}_g\delta^\mu_\nu - \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^a_\lambda}\right)e^\nu_a + \sqrt{-g}T^\mu_a \tag{22}$$

We now define the gravitational energy-momentum as t^μ_a

$$\sqrt{-g}t^\mu_a = \left(\mathcal{L}_g\delta^\mu_\nu - \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^a_\lambda}\right)e^\nu_a + \frac{\partial\mathcal{L}_g}{\partial\partial_\mu e^\nu_b} e^\sigma_b\partial_\sigma e^\nu_a \tag{23}$$

Now let's consider the total energy-momentum tensor, which can be decomposed as [8–11]

$$\Theta^M_a = T^M_a + t^M_a \tag{24}$$

where T_a^M denotes the energy-momentum tensor of matter while t_a^M denotes the energy-momentum tensor of the gravitational field. From now on the lowercase Latin letter a is the index of the non-coordinate bases from the orthonormal frame. Notice that in this formulation the energy-momentum tensors are vector-valued 1-forms on a Riemannian manifold, thus the conservation of energy-momentum can be written in a covariant form from the general conservation law (21)

$$\nabla_M \Theta_a^M = \frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} \Theta_a^M) = 0 \tag{25}$$

It is shown that there exists a superpotential

$$V_a^{MN} = \frac{\partial \mathcal{L}_g}{\partial(\partial_M e_b^P)} e_b^N e_a^P \quad V_a^{MN} = -V_a^{NM} \tag{26}$$

where e_b^M denotes the vielbeins, and $\mathcal{L}_g$ is the Lagrangian of the gravity. We can prove that the total energy-momentum tensor can be expressed by the superpotential

$$\sqrt{-g}\ \Theta_a^M = \partial_N V_a^{MN} \tag{27}$$

so that the conservation of energy-momentum can be expressed as

$$\partial_M(\partial_N V_a^{MN}) = 0 \tag{28}$$

Then we can calculate the usual energy-momentum four-vector through

$$P_a = \int_\Sigma \sqrt{-g} \Theta_a^t d\Sigma = \int_S V_a^{tN} dS_N \tag{29}$$

where the integration hypersurface Σ is defined by $t = constant$, and the second part of the equation is derived from Gauss's law. Also we can calculate the energy-momentum density

$$\varepsilon_a = \sqrt{-g} \Theta_a^t = \partial_N V_a^{tN} \tag{30}$$

Now we can use this formulation to calculate the energy-momentum of our brane-world model. The Lagrangian of gravity here is

$$\mathcal{L}_g = \sqrt{-g}[2M^3 R - \Lambda] - \sqrt{-g^1} \Lambda_1 \delta(y - \pi) - \sqrt{-g^2} \Lambda_2 \delta(y) \tag{31}$$

Using the orthonormal frames, this can be expressed as

$$\begin{aligned} \mathcal{L}_g = \sqrt{-g} \left[2M^3 (\omega_a \omega^a - \omega_{abc} \omega^{abc}) + \frac{2}{\sqrt{-g}} \partial_M\ (e_a^M \omega^a) - \Lambda \right] \\ - \sqrt{-g^1} \Lambda_1 \delta(y - \pi) - \sqrt{-g^2} \Lambda_2 \delta(y) \end{aligned} \tag{32}$$

where ω_{abc} is the Ricci rotation coefficient defined by the spin connection ω_{Mb}^a as $\omega_{abc} = e_a^M \eta_{db} \omega_{Mc}^d$ and has the property $\omega_{abc} = -\omega_{acb}$, while $\omega_a = \eta^{bc} \omega_{bac}$. It can be proven that the divergence term in the above equation can be ignored in the construction of the energy-momentum tensor [8]. The Lagrangian which we really use is then

$$\mathcal{L}_g = 2M^3 \sqrt{-g} (\omega_a \omega^a - \omega_{abc} \omega^{abc}) - \sqrt{-g^1} \Lambda_1 \delta(y - \pi) - \sqrt{-g^2} \Lambda_2 \delta(y) \tag{33}$$

Using (26), we can get the expression of the superpotential

$$V_a^{MN} = 4M^3\sqrt{-g}\left[e_b^M e_c^N \omega_a^{bc} + (e_a^M e_b^N - e_a^N e_b^M)\omega^b\right] \tag{34}$$

Rewrite the metric in (2) using the orthonormal frame

$$ds^2 = -\hat{\theta}^0 \otimes \hat{\theta}^0 + \hat{\theta}^1 \otimes \hat{\theta}^1 + \hat{\theta}^2 \otimes \hat{\theta}^2 + \hat{\theta}^3 \otimes \hat{\theta}^3 + \hat{\theta}^4 \otimes \hat{\theta}^4 \tag{35}$$

we are able to get the basis of the orthonormal frame

$$\hat{\theta}^a = (n(t,y)dt, a(t,y)dx^1, a(t,y)dx^2, a(t,y)dx^3, b(t,y)dy) \tag{36}$$

and their components

$$e_t^0 = n(t,y) \qquad e_{x^i}^i = a(t,y) \qquad e_y^4 = b(t,y) \tag{37}$$

Then from the torsion-free condition

$$d\hat{\theta}^a + \omega_b^a \wedge \hat{\theta}^b = 0 \tag{38}$$

we are able to find the non-vanishing components of ω_a^{bc} and ω_a

$$\begin{aligned}
&\omega_0^{04} = -\omega_0^{40} = \frac{1}{b}\frac{n'}{n} \qquad \omega_4^{04} = -\omega_4^{40} = \frac{1}{n}\frac{\dot{b}}{b} \\
&\omega_i^{i0} = -\omega_i^{0i} = -\frac{1}{n}\frac{\dot{a}}{a} \qquad \omega_i^{i4} = -\omega_i^{4i} = \frac{1}{b}\frac{a'}{a} \\
&\omega_0 = -\frac{1}{n}\left(\frac{\dot{b}}{b} - 3\frac{\dot{a}}{a}\right) \qquad \omega_4 = -\frac{1}{b}\left(\frac{n'}{n} + 3\frac{a'}{a}\right)
\end{aligned} \tag{39}$$

The superpotential can be derived from (34)

$$\begin{aligned}
V_0^{ty} &= -V_0^{yt} = \frac{-12M^3a^2a'}{b} \\
V_i^{x^it} &= -V_i^{tx^i} = \frac{4M^3a^2b}{n}\left(\frac{\dot{b}}{b} - 4\frac{\dot{a}}{a}\right) \\
V_i^{x^iy} &= -V_i^{yx^i} = \frac{4M^3a^2n}{b}\left(-\frac{n'}{n} - 2\frac{a'}{a}\right)
\end{aligned} \tag{40}$$

Now we can calculate the energy-momentum density from (30). We find that the only non-zero component of the energy-momentum density is the energy density

$$\varepsilon_0 = -12M^3\left(\frac{aa'^2}{b} - \frac{a^2a'b'}{b^2} + \frac{a^2a''}{b}\right) \tag{41}$$

which is obviously independent of $n(t,y)$. Therefore, the only non-zero component of the energy-momentum four-vector is the total energy

$$P_0 = 12M^3\mathcal{V}\left(\frac{a^2a'}{b}\right)\Bigg|_{y=\pi}^{y=0} \tag{42}$$

where $\mathcal{V}$ represents the volume of the three dimensional space. We will discuss the implications of these results through analyzing a specific model in the next section.

3 Energy-Momentum of the Cosmological RS Model

A natural generalization of RS model to the cosmological content has inflationary property [17], which include the original RS model as the static limit. Using the notations above, this generalization is obtained if we set

$$a = A(t)E(y) \qquad n = E(y) \qquad b = B_0 \tag{43}$$

where B_0 is a constant which represents the size of the fifth dimension. Then we can solve the Einstein equations to get the expressions of $A(t)$ and $E(y)$. Like in RS model, we only consider the cosmological constants dominated cases, so the Einstein equations are

$$G_{00} = \frac{3}{E^2}\left(\frac{\dot{A}}{A}\right)^2 - \frac{3}{B_0^2}\left[\left(\frac{E'}{E}\right)^2 + \frac{E''}{E}\right] = \frac{1}{4M^3}\left[\Lambda + \frac{\Lambda_1}{B_0}\delta(y) + \frac{\Lambda_2}{B_0}\delta(y-\pi)\right] \tag{44}$$

$$\begin{aligned} G_{ii} &= \frac{1}{E^2}\left[\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\ddot{A}}{A}\right] - \frac{3}{B_0^2}\left[\left(\frac{E'}{E}\right)^2 + \frac{E''}{E}\right] \\ &= \frac{1}{4M^3}\left[\Lambda + \frac{\Lambda_1}{B_0}\delta(y) + \frac{\Lambda_2}{B_0}\delta(y-\pi)\right] \end{aligned} \tag{45}$$

$$G_{44} = \frac{3}{E^2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \frac{\ddot{A}}{A}\right] - \frac{6}{B_0^2}\left(\frac{E'}{E}\right)^2 = \frac{\Lambda}{4M^3} \tag{46}$$

where the (04) part of the Einstein tensor automatically vanishes. The boundary conditions are

$$\begin{aligned} \left.\frac{E'}{E}\right|_{0_-}^{0+} &= -\frac{B_0}{12M^3}\Lambda_1 \\ \left.\frac{E'}{E}\right|_{-\pi}^{+\pi} &= -\frac{B_0}{12M^3}\Lambda_2 \end{aligned} \tag{47}$$

Using (44) and (46), it is shown that there is a inflationary solution of $A(t)$ [17]

$$A(t) = A_0 e^{H_0 t} \qquad H_0 = \frac{\dot{A}}{A} \tag{48}$$

Then from (46) we can find the solution to $E(y)$ satisfying the orbifold symmetry [17]

$$E(y) = \frac{H_0}{k}\sinh(-kB_0|y| + c) \quad k = \sqrt{\frac{-\Lambda}{24M^3}} \tag{49}$$

From the boundary conditions (47) we get

$$k_1 = k\coth(c) \qquad -k_2 = k\coth(-kB_0\pi + c) \tag{50}$$

where $k_i = \Lambda_i/24M^3$. So the metric of this model is

$$ds^2 = \left(\frac{H_0 A_0}{k}\right)^2 \sinh^2(-kB_0|y| + c)[-dt^2 + e^{2H_0 t}\delta_{ij}\, dx^i dx^j] + B_0 dy^2 \tag{51}$$

Obviously this metric belongs to the "warped geometry".

Now we can calculate the energy-momentum density and the energy-momentum four-vector of this model from (41) and (42). Again the only non-zero components are the energy density and total energy at a given time t

$$\varepsilon_0 = \frac{-12M^3 B_0 H_0^3 A_0^3}{k} e^{-3H_0 t} \cosh(-2kB_0|y| + 2c)\sinh(-kB_0|y| + c) \tag{52}$$

$$P_0 = \frac{2H_0^3 A_0^3 M^3}{k^2} e^{-3H_0 t} \mathcal{V}[3\cosh(c) - \cosh(3c) - 3\cosh(c - B_0 k\pi) + \cosh(3c - 3B_0 k\pi)] \tag{53}$$

Let consider the total energy P_0 first. It is shown by many authors that the total energy of a gravitational system is zero (see for example [19, 20]), while from (53) we can see that the total energy is infinity. This might be from the effect of the gravity along the warped fifth dimension. Notice if $B_0 \to 0$, i.e. the fifth dimension vanishes, then we have $P_0 \to 0$, which is consistent with [18].

On the other hand, if we set

$$a = n = e^{-kr|y|} \qquad b = r \tag{54}$$

then we recover the original RS model as the static limit. In this case the energy-momentum density and total energy are [18]

$$\varepsilon_0 = -36M^3 k^2 e^{-3kr|y|} \tag{55}$$

$$P_0 = 12M^3 k(e^{-3kr\pi} - 1)\mathcal{V} \tag{56}$$

If $r \to 0$ then $P_0 \to 0$, which is consistent with above discussion. However, (55) indicates that the energy density, which only includes the energy density of the gravity along the fifth dimension, is much grater at the Planck brane than at the TeV brane

$$\frac{\varepsilon_0^P}{\varepsilon_0^T} = e^{3kr\pi} \tag{57}$$

This might be a reflection of the gauge hierarchy in RS model that the gravity is much stronger on the Planck brane than on the TeV brane, because the exponential difference between the gravitational energy density on the two branes which means gravity is mostly localized on the Planck brane [18]. In RS $e^{kr\pi}$ is required to be of order 10^{15} in order to generate the gauge hierarchy, then we have

$$\frac{\varepsilon_0^P}{\varepsilon_0^T} \approx 10^{45} \tag{58}$$

Notice that the energy density here is negative. This might be the result of the brane model—the total energy of a gravitational system is zero, which means if we set the matter energy to be positive, then the gravitational energy could be negative. In this brane-world model there is no matter energy to cancel the gravitational energy along the fifth dimension so that the energy density here could be pure gravitational energy which is negative.

RS model is only the static limit to our discussion here. Let's consider now the gauge hierarchy from (52). We can see

$$\frac{\varepsilon_0^P}{\varepsilon_0^T} = \frac{\cosh(2c)\sinh(c)}{\cosh(-2kB_0\pi + 2c)\sinh(-kB_0\pi + c)} \tag{59}$$

According to our previous discussion, we can set the energy density here to be negative. This puts a constrain on the constant c that it should be larger that $kB_0\pi$ (which we will meet again later in agreement with the result from [17]), then again the energy density on the Planck brane could be much greater than on the TeV brane. Furthermore, if c is near $kB_0\pi$, we can get a extremely large difference between these two energy densities. Based on our previous discussion, this again means that we find a hierarchy between the Planck brane and the TeV brane if $\varepsilon_0^P/\varepsilon_0^T \approx 10^{45}$. There is only one requirement here in order to generate this hierarchy that c should be larger that $kB_0\pi$, which is natural according to our discussion. The magnitude of the hierarchy is determined by the difference between c and $kB_0\pi$; whatever the value of the size of the fifth dimension is, the suitable hierarchy is generated from choosing the constant c. If B_0 is determined by some other mechanism, then we can decide the value of c according to the gauge hierarchy. Therefore, the gauge hierarchy could be regarded as a general feature of this generalized RS model, no matter how the size of the extra dimension is stabilized.

From (50) we can see that the constrain on c is cast into the constrain on k_1 and k_2, which is consistent with the results from [17], which supports our discussion of setting the energy density to be negative. Our requirement that $c > kB_0\pi$, which gives a negative energy density, agrees with the second one of (50) in order to get a negative k_2. Indeed, the gauge hierarchy problem and the cosmological constant problem are recast into the fine-tuning problem of the bulk and the brane cosmological constants, whose detail can be found in [17], for instance, $k\,\text{csch}(-kB_0\pi + c) = \sqrt{k_2^2 - k^2} \lesssim 10^{-60}M_P$. However, the cosmological constant problem here is dependent upon the value of B_0, i.e. the modulus stabilization, while the gauge hierarchy may not.

To conclude, we analyze the covariant energy-momentum of a general cosmological five-dimensional brane model, then specifically of a generalized RS model which includes the original RS model as the static limit. The total energy in our case agrees with the general result from others, which is that the total energy of a gravitational system is zero, if the extra dimension vanishes. We are also able to show that the gauge hierarchy is a general feature of this model from a gravitational point of view, which is independent from the stabilization of the size of the fifth dimension. Our results confirms the conclusions from [17] but from a different point of view.

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